The effect of gauge conditions on waveforms from binary black hole coalescence

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Abstract

Over the past year and a half, a number of groups have produced stable runs of a binary black hole system evolving through merger and ringdown [1][2][3]. In [2][3], in particular, the tremendous speedup to the field was driven by special sets of gauge evolution equations, capable of handling several issues that have traditionally plagued black hole simulations: avoiding the singularity, guaranteeing a constraint satisfying solution at least in the exterior region, and advecting the holes through the numerical grid.

Since several successful recipes have already been proposed, the goal of this study is to review them and analysize the consistency of the published results. An interesting starting point is, for instance, the comparison of the waveform outcome of each different gauge prescription.

3+1 decompositions in Numerical Relativity

Traditionally, numerical solutions of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

have been carried out by means of 3+1 decompositions, i.e. by choosing a timelike direction in the spacetime and a corresponding foliation into three-dimensional, spacelike surfaces. One such 3+1 separation is known as the Arnowitt-Deser-Misner (ADM) decomposition, in which the line element takes on the general form:

$$ds^2 = -\alpha^2 dt^2 + h_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

where α is referred to as the lapse, and the vector β^i as the shift. The fundamental evolved variables are the spatial metric h_{ij} and the extrinsic curvature K_{ij}, defined as:

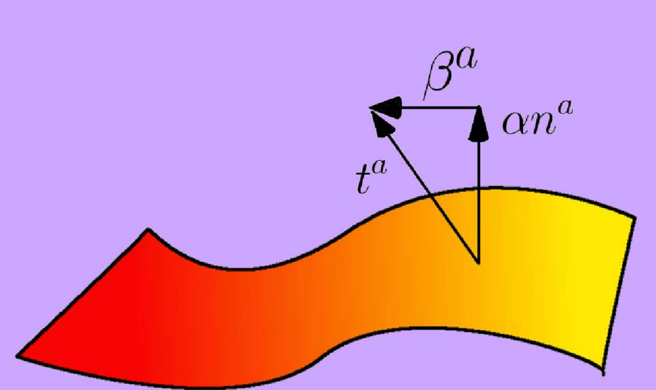
$$K_{ij} = -h_i^k h_j^h \nabla_{(k} n_{h)}$$

and $n_h = -\alpha \nabla_h t$.

From the numerical standpoint, the ADM equations have not been very successful. Several alternative reformulations, both first and second order in time, have been proposed; one such alternative, due to Baumgarte, Shapiro, Shibata and Nakamura, has proved to be particularly stable and is now widely used. In this decomposition, the fundamental variables to evolve are the conformal spatial metric $\tilde{h}_{ij}$, the trace of the extrinsic curvature K, the traceless conformal curvature $\tilde{A}_{ij}$, the conformal factor Φ and the conformal connection functions $\tilde{\Gamma}^i$, related to the ADM variables by:

$$\begin{aligned} \Phi &= \frac{1}{12} \ln h \\ \tilde{h}_{ij} &= e^{-4\Phi} h_{ij} \\ \tilde{A}_{ij} &= e^{-4\Phi}(K_{ij} - \frac{1}{3} h_{ij} K) \\ \tilde{\Gamma}^i &= \tilde{h}^{jk} \tilde{\Gamma}^i_{jk} \end{aligned}$$

Since its introduction, the BSSN decomposition of Einstein's equations has proved to possess the right features to ensure long-lived, constraint-preserving evolutions of black hole systems.



However, the recent breakthroughs in binary black hole simulations have shown that the choice of the lapse and shift functions is crucial in order to ensure stability and convergence of the runs. In particular, several variations of the classic parabolic 1+log and hyperbolic $\Gamma-$driver conditions [4] have been applied, with appropriate modifications.

The purpose of this poster is to check the consistency of the different gauge predictions by running all the different binary simulations in the homogeneous framework provided by the PSU code and comparing the resulting waveforms.

Initial data

In this poster, we study the effects of the gauge choice for a plunging binary black hole system, starting out from the last stable quasi-equilibrium configuration, known as QC-0. The sequence of stable orbits is found by imposing a stationariety condition for the binding energy E_b, defined as the difference between the ADM mass of the spacetime and the sum of the masses of the black holes:

$$\frac{\partial E_b}{\partial d} := \frac{\partial\,[M_{ADM} - 2M]}{\partial d} = 0$$

where d is the separation between the holes.

The last stable orbit denoted as QC-0 is the saddle point between the binding energy stationary points that minimize E_b and those that maximize it:

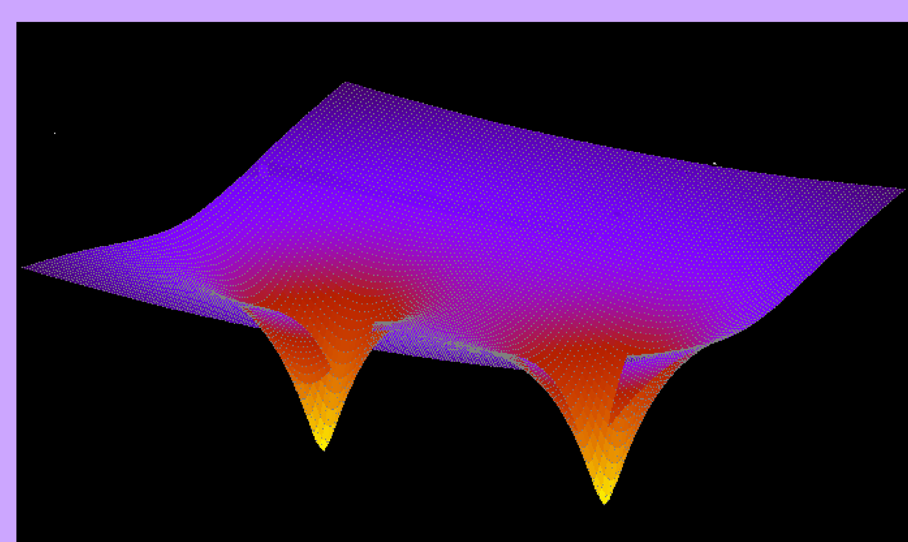
$$\frac{\partial^2 E_b}{\partial d^2} = 0$$

The two holes start approximately 2.5M apart and merge after less than one orbit.

Binary black hole runs in the PSU code

The PSU code is a `Cactus` based algorithm using fixed mesh refinement (through the `Carpet` package), and implementing the BSSN system (in its variant due to Yo [5]) via the `Kranc` code generator [6].

Current binary runs are performed using Marcus Ansorg's initial data generator `TwoPunctures` and a number of analysis tools including AEI's apparent horizon tracker `AHFinderDirect` and the Zerilli wave extraction thorn `WaveExtract`.



Initial lapse profile for the binary runs.

We have used this system to test the impact of three currently reported gauge choices on the physical output of the runs. The simulations involve a binary black hole system starting from a QC-0 quasi-equilibrium configuration, with outer boundaries at $64M$, six refinement levels and bitant symmetry.

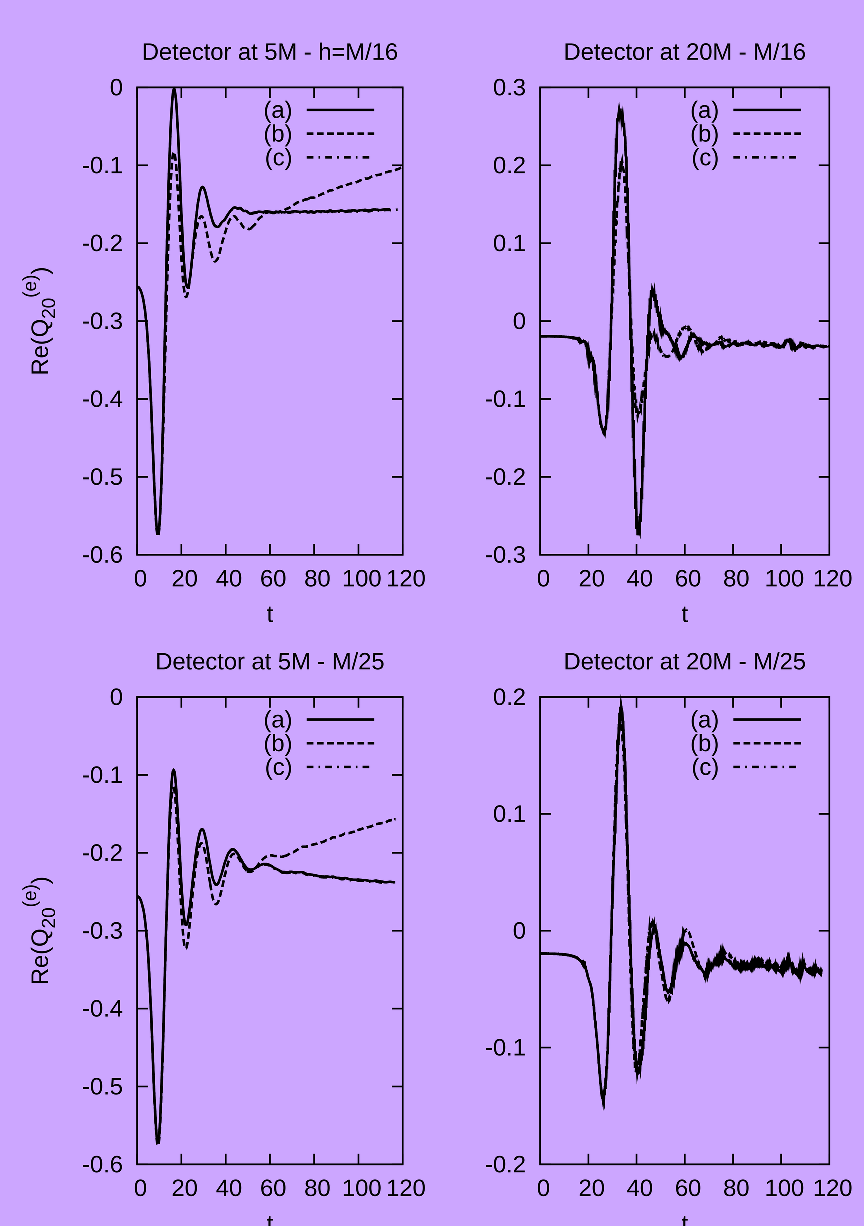
The three gauge choices are as follows:

$$\text{(a)[2]} \begin{cases} \partial_t \alpha = -2\alpha K + \beta^i \partial_i \alpha \\ \partial_t \beta^i = \frac{3}{4} B^i \\ \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i \end{cases}$$

$$\text{(b)[3]} \begin{cases} \partial_t \alpha = -2\alpha K + \beta^i \partial_i \alpha \\ \partial_t \beta^i = \frac{3}{4} \alpha B^i \\ \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i - \beta^j \partial_j \tilde{\Gamma}^i \end{cases}$$

$$\text{(c)[7]} \begin{cases} \partial_t \alpha = -2\alpha K + \beta^i \partial_i \alpha \\ \partial_t \beta^i = \frac{3}{4} B^i + \beta^j \partial_j \beta^i \\ \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i + \beta^j \partial_j (B^i - \tilde{\Gamma}^i) \end{cases}$$

We show the real part of the Moncrief Q function (mode $l = 2, m = 0$) for two different resolutions and two different detector locations.



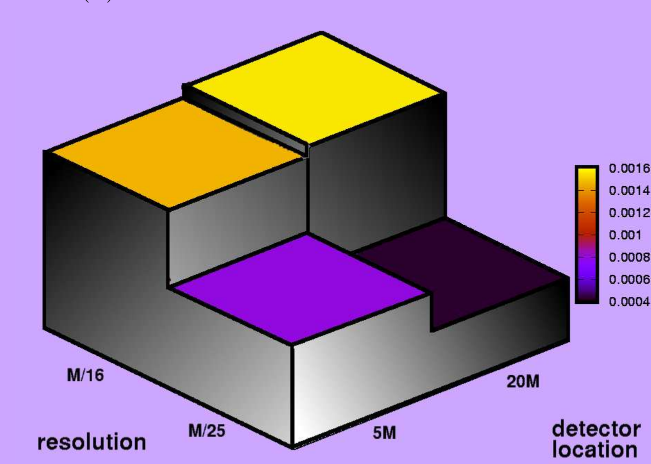
As expected, the agreement between the different predictions improves with increasing resolution and at increasing distance from the merger region.

Error Budget

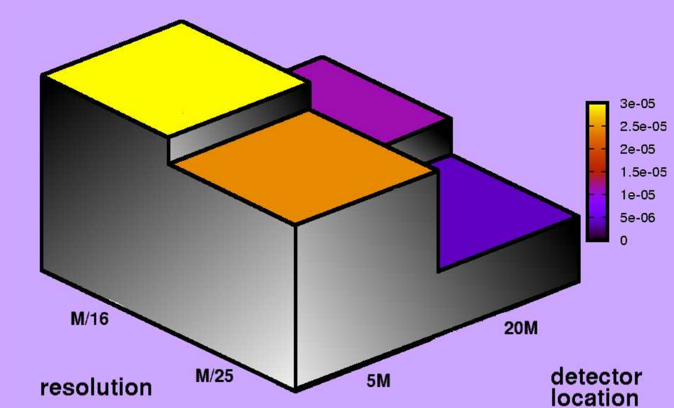
We discuss two possible sources of error in the comparison:

- **Resolution:** the differences could be induced by a different effect of the grid spacing h on each gauge. An appropriate comparison should therefore confront the limit of the three gauge conditions for $h \to 0$, and not the results at finite resolution.
- **Deviations from geodesicity:** the same coordinate label is affixed to different spacetime events in different runs, due precisely to gauge effects. The detectors closer to the merger region are more affected by this deviation, whereas those far away have nearly geodetical worldlines.

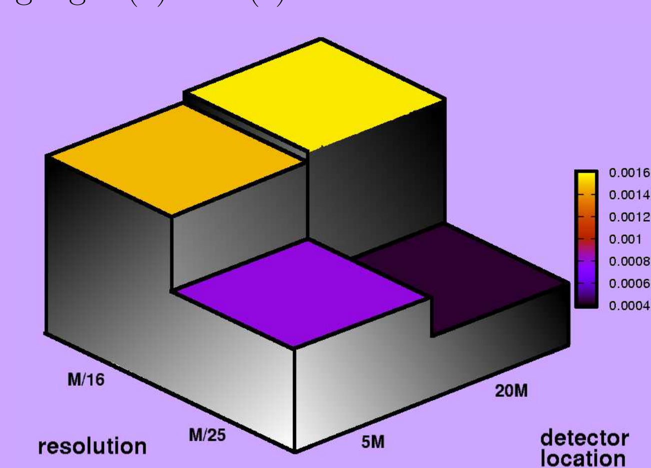
The plots of the norm of the difference between the waveform predictions of -respectively- gauge (a) and (b), (a) and (c) and (b) and (c) are shown below.



Norm of the difference in waveform between gauges (a) and (b).



Norm of the difference in waveform between gauges (a) and (c).



Norm of the difference in waveform between gauges (b) and (c).

Conclusions

This preliminary study shows that there are excellent reasons to believe that the Numerical Relativity community finally possesses a consistent framework for the evolution of binary systems from QC-0, quasi-equilibrium initial data. Waveform predictions from different gauge conditions prove to converge, modulo the numerical errors due to finite resolution and to lack of perfect geodesicity of the detectors.

A rigorous treatment of this problem will have to involve proper convergence tests and large scale simulations with clean waveforms out to very large radii.

An interesting extension will involve the evolution of initial data describing larger separation binaries.

References

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